FLOW OF A VISCOUS GAS IN A SHOCK LAYER WITH EQUILIBRIUM CHEMICAL REACTIONS

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Steady flow of supersonic air over a sphere is examined, allowing for viscosity, heat conduction, and actual physical and chemical processes. Flow in the shock layer at flight speeds in the range 3 km/sec $\leq V_\infty \leq 10$ km/sec ($10^4 \leq R_\infty \leq 10^6$) is investigated, under the assumption of local thermodynamic equilibrium. The flow is described by simplified Navier-Stokes equations, which are solved by a finite difference method. The case of a cooled surface is examined. The distribution of gasdynamic parameters is obtained in different flow regimes. The distribution of heat flux and friction coefficient is investigated as a function of the oncoming-stream parameters and the sphere radius. The shape and position of the shock wave are determined, and the stream lines and sonic lines are constructed.

For a gas with constant specific heat, at Reynolds numbers $R_\infty \leq 10^3$, supersonic flow over blunt bodies has been considered in terms of the simplified and the full Navier-Stokes equations in [1-4].

1. We consider flow in the region ABCD (Fig. 1) enclosed between the detached shock, the body surface, the axis of symmetry, and the surface π. The surface π is chosen so that the downstream flow should not appreciably affect the gas parameters in the region ABCD. The simplified Navier-Stokes equations used, given in [1], contain the full terms of the gasdynamic equations of an inviscid flow and the boundary layer equations.

All quantities are nondimensionalized as follows (primes denote dimensionless quantities):

$$x' = \frac{x}{r_0}, \quad y' = \frac{y}{r_0}, \quad r' = \frac{r}{r_0}, \quad \theta = \frac{s}{r_0}$$

$$\varepsilon' = \frac{\varepsilon}{r_0}, \quad v' = \frac{v}{V_m}, \quad u' = \frac{u}{V_m}, \quad V_m^2 = V_\infty^2 + 2h_\infty$$

$$h' = \frac{h}{{}^1/{}_2 V_m^2}, \quad \rho' = \frac{\rho}{\rho_\infty}, \quad p' = \frac{p}{\rho_\infty V_m^2}, \quad T' = \frac{T}{(m_\infty / R^\circ) V_m^2}$$

$$\mu' = \frac{\mu}{\mu_s}, \quad \lambda' = \frac{\lambda}{\lambda_s}, \quad R = \frac{V_m r_0 \rho_\infty}{\mu_s}, \quad P = \frac{R^\circ \mu_s}{m_\infty \lambda_s}$$

Here x and y are rectangular coordinates, x directed upstream; r and θ are spherical polar coordinates; u and v are the r and θ components of velocity V, respectively; ε is the shock-wave standoff distance; h is the enthalpy; ρ is the density; p is the pressure; T is the temperature; V_m is the maximum velocity; r_0 is the sphere radius; m_∞ is the molecular weight; R° is the universal gas constant; μ is the dynamic viscosity; λ is the total heat conduction; R is the Reynolds number; P is the Prandtl scale number; μ_s, λ_s are the values of μ and λ behind the shock, on the axis of symmetry; the subscript ∞ denotes values of the parameters in the oncoming stream.

In dimensionless variables the original system of equations has the form (primes on the dimensionless quantities are omitted)

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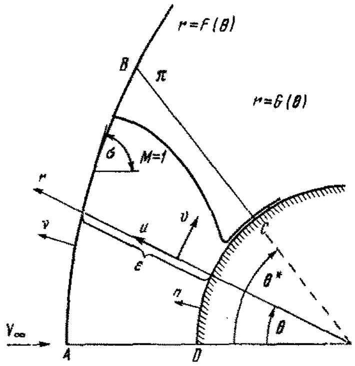

Fig. 1

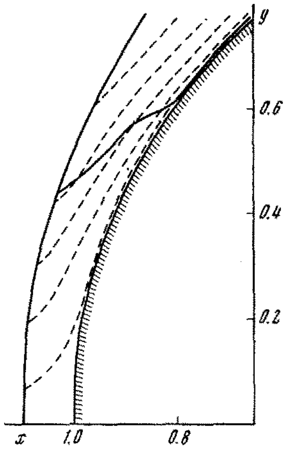

Fig. 2

$$
\begin{gathered}
r\frac{\partial}{\partial r}(\rho u)+\frac{\partial}{\partial \theta}(\rho v)+\rho v \operatorname{ctg}\theta+2\rho u=0\\
r\rho u\frac{\partial u}{\partial r}+\rho v\frac{\partial u}{\partial \theta}-\rho v^2=-r\frac{\partial p}{\partial r}+\frac{4}{3R}\frac{\partial}{\partial r}\left(\mu r\frac{\partial u}{\partial r}\right)\\
-\frac{2}{3R}\frac{\partial}{\partial r}\left(\mu\frac{\partial v}{\partial \theta}\right)-\frac{2}{3R}\frac{\partial}{\partial r}(\mu v)\operatorname{ctg}\theta+\frac{1}{R}\frac{\partial}{\partial \theta}\left(\mu\frac{\partial v}{\partial r}\right)+\frac{\mu}{R}\frac{\partial v}{\partial r}\operatorname{ctg}\theta\\
r\rho u\frac{\partial v}{\partial r}+\rho v\frac{\partial v}{\partial \theta}+\rho uv=-\frac{\partial p}{\partial \theta}+\frac{1}{R}\frac{\partial}{\partial r}\left(\mu r\frac{\partial v}{\partial r}\right)-\frac{v}{R}\frac{\partial \mu}{\partial r}+\frac{\mu}{R}\frac{\partial v}{\partial r}\\
\frac{r\rho u}{2}\frac{\partial h}{\partial r}+\frac{\rho v}{2}\frac{\partial h}{\partial \theta}=ru\frac{\partial p}{\partial r}+v\frac{\partial p}{\partial \theta}+\frac{r\mu}{R}\left[r\frac{\partial}{\partial r}\left(\frac{v}{r}\right)\right]^2+\frac{1}{RP}\left[\frac{\partial}{\partial r}\left(\lambda r\frac{\partial T}{\partial r}\right)+\lambda\frac{\partial T}{\partial r}\right]\\
\rho=\rho(p,\ T),\qquad h=h(p,\ T)
\end{gathered}
\tag{1.1}
$$

For the numerical solution it is convenient to convert to the new independent variables

$$\xi=\frac{r-G(\theta)}{F(\theta)-G(\theta)},\quad \zeta=\theta \tag{1.2}$$

and to compress the coordinate lines ξ =const towards the body surface by means of the transformation

$$z=\frac{\ln(1+\sqrt{R}\,\xi)}{\ln(1+\sqrt{R})} \tag{1.3}$$

The boundary conditions become as follows. The bow shock is considered as a surface of separation, the conditions at it are determined by the Rankine-Hugoniot relations, and its location is found during the solution. Symmetry conditions are used on the axis $\theta=0$. The conditions at the body surface take the form

$$u=0,\qquad v=0,\qquad T=\text{const} \tag{1.4}$$

The body surface temperature is assumed to be 2000°K.

The approximations of [5] were used for the thermodynamic functions of air. The viscosity and the total thermal conductivity were taken from [6].

The solution was obtained by a finite difference method, using a nine-point scheme. The system of difference equations, undetermined at the as-yet-unknown location of the shock, was closed by using at the body surface a projection of the momentum equation along the z axis. The nonlinear system of difference equations was solved by Newton's method.

The calculations were performed in the following range of initial conditions:

$$10\leqslant M_\infty\leqslant 34,\ 0.0002\ \text{atm}\leqslant p_\infty\leqslant 0.004\,\text{atm}\quad(10^4\leqslant R\leqslant 10^5)$$

This determined the shock standoff distance, the gasdynamic parameters in the shock layer, the friction stress τ, and the heat flux q to the body surface:

$$
\begin{gathered}
c_f=\frac{\tau}{{}^1\!/\!_2\rho_\infty V_m{}^2},\qquad \tau=\mu\left.\frac{\partial v}{\partial r}\right|_{r=r_0}\\
q'=\frac{q}{\rho_\infty V_m{}^3},\qquad q=\lambda\left.\frac{\partial T}{\partial r}\right|_{r=r_0}
\end{gathered}
$$

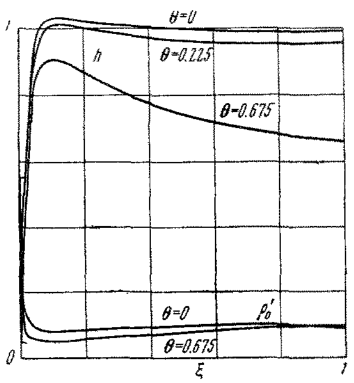

Fig. 3

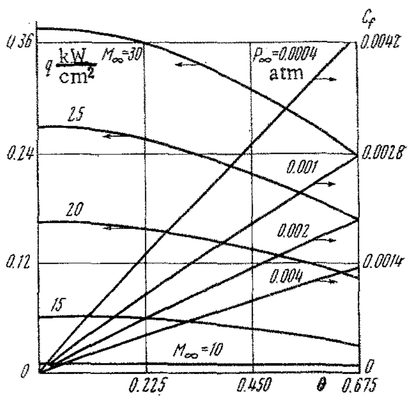

Fig. 4

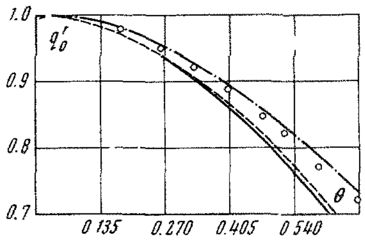

Fig. 5

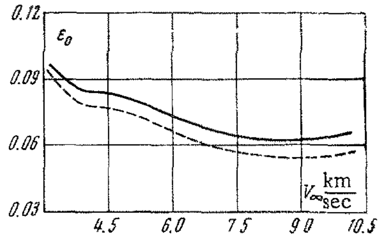

Fig. 6

2. Figure 2 shows the locations of the shock, the sonic line (solid line), and the streamlines (broken lines) for $M_\infty = 10$, $T_\infty = 250°K$, $p_\infty = 0.001$ atm. Figure 3 shows the distributions of enthalpy h and reduced density $\rho_0' = \rho/\rho_0$ across the shock layer (ρ_0 is the density at the stagnation point) for $M_\infty = 33.75$, $T_\infty = 240.6°K$, $p_\infty = 0.0002234$ atm, and various values of the longitudinal coordinate θ. For $\xi \approx 0.1$ the enthalpy has a maximum and the density a minimum. This nonmonotonic nature becomes more pronounced with increase of θ. Figure 4 shows the variation of friction factor c_f ($M_\infty = 20$, $T_\infty = 250°K$) and heat flux q ($p_\infty = 0.001$ atm, $T_\infty = 250°K$) along the surface of the sphere $r_0 = 1.5$ m. It can be seen that the heat flux decreases with decrease of M_∞, and its profile becomes nearly linear. The solid line of Fig. 5 shows the standoff distance along the axis of symmetry ε_0 as a function of the free-stream velocity V_∞ ($p_\infty = 0.001$ atm). The broken line shows the calculations for an inviscid gas by scheme II of the method of integral relations [2]. Allowance for the viscosity and the heat conduction of the gas leads to an increase of 10–12% in the standoff distance in the velocity range considered. Figure 6 shows the distribution of reduced heat flux $q_0' = q/q_0$ (q_0 is the flux at the stagnation point) along the sphere surface and a comparison with results from boundary layer theory. It should be noted that the heat-flux values for the different Mach numbers $10 \le M_\infty \le 30$ and various pressures in the free stream $0.0004 \text{ atm} \le p_\infty \le 0.004$ atm practically coincide on a single curve (solid line), confirming the conclusion reached in [7] that the heat-flux distribution over a sphere is universal for $M_\infty > 10$ and $h_0 \ll h_S$. The dotted line shows the approximation of [7]

$$q_0' = 0.55 + 0.45 \cos 2\theta$$

The dot-dash line shows the results of [8], calculated in the approximation of local self-similarity of the boundary layer equations. The points are the experimental data of [8].

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